1. Resolver, pelo emprego de determinantes, o sistema de equações:

$$\begin{cases} 3x + y + z = 4 \\ x - y + 2z = 6 \\ x + 2y - z = -3 \end{cases}$$

$$D = \left|\begin{array}{ccc|cc} 3 & 1 & 1 & 3 & 1 \\ 1 & -1 & 2 & 1 & -1 \\ 1 & 2 & -1 & 1 & 2 \end{array}\right|$$

$$D = 3\times(-1)\times(-1) + 1\times2\times1 + 1\times1\times2 - 1\times(-1)\times1 - 2\times2\times3 - (-1)\times1\times1 = -3$$

$$D_x = \left|\begin{array}{ccc|cc} 4 & 1 & 1 & 4 & 1 \\ 6 & -1 & 2 & 6 & -1 \\ -3 & 2 & -1 & -3 & 2 \end{array}\right|$$

$$D_x = 4\times(-1)\times(-1) + 1\times2\times(-3) + 1\times6\times2 - (-3)\times(-1)\times1 - 2\times2\times4 - (-1)\times6\times1 = -3$$

$$D_y = \left|\begin{array}{ccc|cc} 3 & 4 & 1 & 3 & 4 \\ 1 & 6 & 2 & 1 & 6 \\ 1 & -3 & -1 & 1 & -3 \end{array}\right|$$

$$D_y = 3\times6\times(-1) + 4\times2\times1 + 1\times1\times(-3) - 1\times6\times1 - (-3)\times2\times3 - (-1)\times1\times4 = 3$$

$$D_z = \left|\begin{array}{ccc|cc} 3 & 1 & 4 & 3 & 1 \\ 1 & -1 & 6 & 1 & -1 \\ 1 & 2 & -3 & 1 & 2 \end{array}\right|$$

$$D_z = 3\times(-1)\times(-3) + 1\times6\times1 + 4\times1\times2 - 1\times(-1)\times4 - 2\times6\times3 - (-3)\times1\times1 = -6$$

Solução: $x = \frac{-3}{-3} = 1$ $y = \frac{3}{-3} = -1$ $z = \frac{-6}{-3} = 2$

2. Inverter a matriz: $\begin{bmatrix} 1 & 3 & 3 \\ 2 & 3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$

$$\begin{array}{c} \times(-1)\ \times(-2) \\ \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \Rightarrow \begin{array}{c} \\ \times\left(-\frac{1}{3}\right) \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & -3 & -7 & -2 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ \times(-3) \\ \times 1 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{7}{3} & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right] \Rightarrow \begin{array}{c} \\ \\ \times 3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -4 & -1 & 1 & 0 \\ 0 & 1 & \frac{7}{3} & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 1 \end{array}\right]$$

$$\begin{array}{c} \\ \\ \times 4\ \ \times\left(-\frac{3}{7}\right) \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -4 & -1 & 1 & 0 \\ 0 & 1 & \frac{7}{3} & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -3 & 12 \\ 0 & 1 & 0 & 3 & 2 & -7 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{array}\right]$$

3. Multiplicar a matriz **A** pela matriz **B**: $\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 1 & -5 \\ 2 & 4 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$

$$\underset{(3\times 2)}{\begin{bmatrix} 3 & 2 \\ 1 & -5 \\ 2 & 4 \end{bmatrix}} \times \underset{(2\times 3)}{\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}} = \begin{bmatrix} 3\times 0 + 2\times(-1) & 3\times(-1) + 2\times 0 & 3\times 0 + 2\times(-1) \\ 1\times 0 + (-5)\times(-1) & 1\times(-1) + (-5)\times 0 & 1\times 0 + (-5)\times(-1) \\ 2\times 0 + 4\times(-1) & 2\times(-1) + 4\times 0 & 2\times 0 + 4\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 & -2 \\ 5 & -1 & 5 \\ -4 & -2 & -4 \end{bmatrix}$$

$(3\times 2)\times(2\times 3) = (3\times 3)$

4. Determinar a derivada das seguintes expressões:

a)

$$y = (x+3)\cdot\sqrt{x}$$

$$u = x+3 \longrightarrow u' = 1$$

$$v = \sqrt{x} = x^{\frac{1}{2}} \Longrightarrow v' = \frac{1}{2}\,x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$y' = 1\times\sqrt{x} + (x+3)\times\frac{1}{2\sqrt{x}} = \underset{(2\sqrt{x})}{\sqrt{x}} + \frac{x+3}{2\cdot\sqrt{x}} = \frac{2x+x+3}{2\cdot\sqrt{x}} = \frac{3x+3}{2\cdot\sqrt{x}}$$

b)

$$y = \frac{4x}{(x^2-1)^2}$$

$$u = 4x \Longrightarrow u' = 4$$

$$v = (x^2-1)^2 \Longrightarrow v' = 2\times(x^2-1)\times 2x = 4x\,(x^2-1)$$

$$y' = \frac{4\times(x^2-1)^2 - 4x\times 4x\,(x^2-1)}{(x^2-1)^4} = \frac{(x^2-1)\left(4\cdot(x^2-1) - 16x^2\right)}{(x^2-1)^4}$$

$$= \frac{4x^2-4-16x^2}{(x^2-1)^3} = -\frac{4+12x^2}{(x^2-1)^3} = -4\cdot\frac{1+3x^2}{(x^2-1)^3}$$

c)

$$y = (x+1)\cdot(2x-2)\cdot(3x+4)$$

$$u = (x+1)\cdot(2x-2) \Longrightarrow u' = 1\times(2x-2) + (x+1)\times 2 = 4x$$

$$v = 3x+4 \Longrightarrow v' = 3$$

$$y' = 4x\cdot(3x+4) + (x+1)\cdot(2x-2)\times 3 = 12x^2 + 16x + 6x^2 - 6x + 6x - 6$$

$$= 18x^2 + 16x - 6$$

ou

$$y=(x+1)\cdot(2x-2)\cdot(3x+4) = (2x^2-2x+2x-2)\cdot(3x+4)$$

$$= (2x^2-2)\cdot(3x+4) = 6x^3+8x^2-6x-8$$

$$y' = 6\times 3\cdot x^2\times 1 + 8\times 2\times x\times 1 - 6\times 1 = 18x^2+16x-6$$

ou

$$y=(x+1)(2x-2)(3x+4) = (x+1)\times 2\times(x-1)\cdot(3x+4) = 2\cdot(x^2-1)\cdot(3x+4)$$

$$u = x^2-1 \Rightarrow u' = 2x \qquad v = 3x+4 \Rightarrow v' = 3$$

$$y' = 2\times\left(2x\cdot(3x+4) + (x^2-1)\times 3\right) = 2\times\left(6x^2+8x+3x^2-3\right)$$
$$= 18x^2+16x-6$$

d)

$$y = \sqrt[3]{4x+\frac{1}{x^5}} = \left(4x+\frac{1}{x^5}\right)^{\frac{1}{3}}$$

$$u = 4x+\frac{1}{x^5} = 4x + x^{-5} \Rightarrow u' = 4-5\cdot x^{-6} = 4-\frac{5}{x^6}$$

$$y' = \frac{1}{3}\left(4x+\frac{1}{x^5}\right)^{-\frac{2}{3}}\times\left(4-\frac{5}{x^6}\right) = \frac{4-\frac{5}{x^6}}{3\cdot\sqrt[3]{\left(4x+\frac{1}{x^5}\right)^2}}$$

5. Determinar y' dada a equação $2x^2y - xy^2 = 4$

$$2\cdot x^2\cdot y - x\cdot y^2 = 4 \Rightarrow (2x^2\cdot y)' - (x\cdot y^2)' = (4)'$$

$$4xy + 2\cdot x^2 y' - y^2 - 2\cdot x\cdot y\cdot y' = 0 \Rightarrow$$

$$2x^2y' - 2xyy' = y^2 - 4\cdot xy \Rightarrow y' = \frac{y^2-4xy}{2x^2-2xy} = \frac{y(y-4x)}{2x(x-y)}$$

6. Determinar o ponto crítico de $y = 3\sqrt{x} - 2x + 4$, indicando se se trata de um máximo ou de um mínimo.

$$y = 3\sqrt{x} - 2x + 4 = 3x^{\frac{1}{2}} - 2x + 4$$

$$y' = 3 \times \frac{1}{2} \cdot x^{-\frac{1}{2}} - 2 = \frac{3}{2\sqrt{x}} - 2$$

$$y' = 0 \Rightarrow \frac{3}{2\sqrt{x}} - 2 = 0 \Rightarrow \frac{1}{\sqrt{x}} = \frac{4}{3} \Rightarrow \sqrt{x} = \frac{3}{4} \Rightarrow x = \frac{9}{16} \text{ Ponto crítico}$$

Para $x = \frac{8}{16} < \frac{9}{16}$ $\quad y'\left(\frac{8}{16}\right) = \frac{3}{2\sqrt{\frac{8}{16}}} - 2 = 0,12 > 0$

Para $x = \frac{10}{16} > \frac{9}{16}$ $\quad y'\left(\frac{10}{16}\right) = \frac{3}{2\sqrt{\frac{10}{16}}} - 2 = -0,10 < 0$

A derivada passa de positiva a negativa, logo $x = \frac{9}{16}$ é ponto de máximo.