

1. Determinar a derivada das seguintes expressões:

a)

$$y = (x+3) \cdot \sqrt{x}$$

$$u = x+3 \Rightarrow u' = 1$$

$$v = \sqrt{x} = x^{\frac{1}{2}} \Rightarrow v' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$y' = 1 \times \sqrt{x} + (x+3) \times \frac{1}{2\sqrt{x}} = \sqrt{x} + \frac{x+3}{2\sqrt{x}} = \frac{2x + x+3}{2\sqrt{x}} = \frac{3x+3}{2\sqrt{x}}$$

b)

$$y = \frac{-7}{x+3} = -7 \cdot (x+3)^{-1}$$

$$y' = -7 \times (-1) \times (x+3)^{-2} \times (x+3)' = 7 \times (x+3)^{-2} \times 1 = \frac{7}{(x+3)^2}$$

ou

$$u = -7 \Rightarrow u' = 0$$

$$v = x+3 \Rightarrow v' = 1$$

$$y' = \frac{0 \times (x+3) - (-7) \times 1}{(x+3)^2} = \frac{7}{(x+3)^2}$$

c)

$$y = \frac{x+1}{x-1}$$

$$u = x+1 \Rightarrow u' = 1$$

$$v = x-1 \Rightarrow v' = 1$$

$$y' = \frac{1 \times (x-1) - (x+1) \times 1}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

d)

$$y = (-3x^2 + 5x - 1)^4$$

$$u = -3x^2 + 5x - 1 \Rightarrow u' = -6x + 5$$

$$y' = 4 \times (-3x^2 + 5x - 1)^3 \times (-6x + 5)$$

e)

$$y = \frac{x^2 + 1}{x^2 - 1}$$

$$u = x^2 + 1 \Rightarrow u' = 2x$$

$$v = x^2 - 1 \Rightarrow v' = 2x$$

$$y' = \frac{2x \cdot (x^2 - 1) - (x^2 + 1) \cdot 2x}{(x^2 - 1)^2} = \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2 - 1)^2} = \frac{-4x}{(x^2 - 1)^2}$$

f)

$$y = \frac{4x}{(x^2 - 1)^2}$$

$$u = 4x \Rightarrow u' = 4$$

$$v = (x^2 - 1)^2 \Rightarrow v' = 2 \cdot (x^2 - 1) \cdot 2x = 4x(x^2 - 1)$$

$$y' = \frac{4x(x^2 - 1)^2 - 4x \cdot 4x(x^2 - 1)}{(x^2 - 1)^4} = \frac{(x^2 - 1)(4(x^2 - 1) - 16x^2)}{(x^2 - 1)^4}$$

$$= \frac{4x^2 - 4 - 16x^2}{(x^2 - 1)^3} = -\frac{4 + 12x^2}{(x^2 - 1)^3} = -4 \cdot \frac{1 + 3x^2}{(x^2 - 1)^3}$$

g)

$$y = \left(x^2 - \frac{2}{x^3}\right)^2$$

$$u = x^2 - \frac{2}{x^3} = x^2 - 2 \cdot x^{-3}$$

$$u' = 2x - 2x(-3)x^{-4} = 2x + \frac{6}{x^4}$$

$$y' = 2 \cdot \left(x^2 - \frac{2}{x^3}\right) \cdot \left(2x + \frac{6}{x^4}\right)$$

h)

$$y = (x+1) \cdot (2x-2) \cdot (3x+4)$$

$$u = (x+1) \cdot (2x-2) \Rightarrow u' = 1 \cdot (2x-2) + (x+1) \cdot 2 = 4x$$

$$v = 3x+4 \Rightarrow v' = 3$$

$$y' = 4x \cdot (3x+4) + (x+1) \cdot (2x-2) \cdot 3 = 12x^2 + 16x + 6x^2 - 6x + 6x - 6$$

$$= 18x^2 + 16x - 6$$

04

$$y = (x+1) \cdot (2x-2) \cdot (3x+4) = (2x^2 - 2x + 2x - 2) \cdot (3x+4) \\ = (2x^2 - 2) \cdot (3x+4) = 6x^3 + 8x^2 - 6x - 8$$

$$y' = 6 \times 3 \cdot x^2 \times 1 + 8 \times 2 \times x \times 1 - 6 \times 1 = 18x^2 + 16x - 6$$

04

$$y = (x+1)(2x-2)(3x+4) = (x+1) \times 2 \times (x-1) \cdot (3x+4) = 2 \cdot (x^2-1) \cdot (3x+4)$$

$$u = x^2 - 1 \Rightarrow u' = 2x \quad v = 3x + 4 \Rightarrow v' = 3$$

$$y' = 2 \times (2x \cdot (3x+4) + (x^2-1) \times 3) = 2 \times (6x^2 + 8x + 3x^2 - 3) \\ = 18x^2 + 16x - 6$$

i)

$$y = \sqrt[3]{4x + \frac{1}{x^5}} = \left(4x + \frac{1}{x^5}\right)^{\frac{1}{3}}$$

$$u = 4x + \frac{1}{x^5} = 4x + x^{-5} \Rightarrow u' = 4 - 5 \cdot x^{-6} = 4 - \frac{5}{x^6}$$

$$y' = \frac{1}{3} \cdot \left(4x + \frac{1}{x^5}\right)^{-\frac{2}{3}} \times \left(4 - \frac{5}{x^6}\right) = \frac{4 - \frac{5}{x^6}}{3 \cdot \sqrt[3]{\left(4x + \frac{1}{x^5}\right)^2}}$$

2. Determinar  $y'$  dada a equação  $2x^2y - xy^2 = 4$

$$2 \cdot x^2 \cdot y - x \cdot y^2 = 4 \quad \Rightarrow \quad (2x^2 \cdot y)' - (x \cdot y^2)' = (4)'$$

$$4xy + 2 \cdot x^2 y' - y^2 - 2 \cdot x \cdot y \cdot y' = 0 \Rightarrow$$

$$2x^2 y' - 2xy y' = y^2 - 4xy \Rightarrow y' = \frac{y^2 - 4xy}{2x^2 - 2xy} = \frac{y(y-4x)}{2x(x-y)}$$

3. Determinar  $y'$  dada a equação  $x = y \cdot (1 - y^2)$

$$x = y \cdot (1 - y^2) \quad \frac{dx}{dy} = 1 \times (1 - y^2) + y \times (-2)y = 1 - y^2 - 2y^2 = 1 - 3y^2$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{1 - 3y^2}$$

4. Determinar o ponto crítico de  $y = 3\sqrt{x} - 2x + 4$ , indicando se se trata de um máximo ou de um mínimo.

$$y = 3\sqrt{x} - 2x + 4 = 3x^{\frac{1}{2}} - 2x + 4$$

$$y' = 3 \times \frac{1}{2} x^{-\frac{1}{2}} - 2 = \frac{3}{2\sqrt{x}} - 2$$

$$y' = 0 \Rightarrow \frac{3}{2\sqrt{x}} - 2 = 0 \Rightarrow \frac{1}{\sqrt{x}} = \frac{4}{3} \Rightarrow \sqrt{x} = \frac{3}{4} \Rightarrow x = \frac{9}{16} \quad \text{Ponto crítico}$$

$$\text{Para } x = \frac{8}{16} < \frac{9}{16} \quad y'\left(\frac{8}{16}\right) = \frac{3}{2\sqrt{\frac{8}{16}}} - 2 = 0,12 > 0$$

$$\text{Para } x = \frac{10}{16} > \frac{9}{16} \quad y'\left(\frac{10}{16}\right) = \frac{3}{2\sqrt{\frac{10}{16}}} - 2 = -0,10 < 0$$

A derivada passa de positiva a negativa, logo  $x = \frac{9}{16}$  é ponto de máximo.