

1. Resolver, pelo emprego de determinantes, os sistemas de equações:

a)

$$\begin{cases} 2x + 3y = 8 \\ 5x + 4y = 13 \end{cases} \quad D = \begin{vmatrix} 2 & 3 \\ 5 & 4 \end{vmatrix} = 2 \times 4 - 5 \times 3 = -7$$

$$D_x = \begin{vmatrix} 8 & 3 \\ 13 & 4 \end{vmatrix} = 8 \times 4 - 13 \times 3 = -7 \quad x = \frac{-7}{-7} = 1$$

$$D_y = \begin{vmatrix} 2 & 8 \\ 5 & 13 \end{vmatrix} = 2 \times 13 - 5 \times 8 = -14 \quad y = \frac{-14}{-7} = 2$$

b)

$$\begin{cases} 3x + y + z = 4 \\ x - y + 2z = 6 \\ x + 2y - z = -3 \end{cases} \quad D = \begin{vmatrix} 3 & 1 & 1 & | & 4 & 1 \\ 1 & -1 & 2 & | & 6 & -1 \\ 1 & 2 & -1 & | & -3 & 2 \end{vmatrix}$$

$$D = 3 \times (-1) \times (-1) + 1 \times 2 \times 1 + 1 \times 1 \times 2 - 1 \times (-1) \times 1 - 2 \times 2 \times 3 - (-1) \times 1 \times 1 = -3$$

$$D_x = \begin{vmatrix} 4 & 1 & 1 & | & 4 & 1 \\ 6 & -1 & 2 & | & 6 & -1 \\ -3 & 2 & -1 & | & -3 & 2 \end{vmatrix}$$

$$D_x = 4 \times (-1) \times (-1) + 1 \times 2 \times (-3) + 1 \times 6 \times 2 - (-3) \times (-1) \times 1 - 2 \times 2 \times 4 - (-1) \times 6 \times 1 = -3$$

$$D_y = \begin{vmatrix} 3 & 4 & 1 & | & 3 & 4 \\ 1 & 6 & 2 & | & 1 & 6 \\ 1 & -3 & -1 & | & 1 & -3 \end{vmatrix}$$

$$D_y = 3 \times 6 \times (-1) + 4 \times 2 \times 1 + 1 \times 1 \times (-3) - 1 \times 6 \times 1 - (-3) \times 2 \times 3 - (-1) \times 1 \times 4 = 3$$

$$D_z = \begin{vmatrix} 3 & 1 & 4 & | & -3 & 1 \\ 1 & -1 & 6 & | & 1 & -1 \\ 1 & 2 & -3 & | & 1 & 2 \end{vmatrix}$$

$$D_z = 3 \times (-1) \times (-3) + 1 \times 6 \times 1 + 4 \times 1 \times 2 - 1 \times (-1) \times 4 - 2 \times 6 \times 3 - (-3) \times 1 \times 1 = -6$$

Solução: $x = \frac{-3}{-3} = 1$ $y = \frac{3}{-3} = -1$ $z = \frac{-6}{-3} = 2$

2. Determinar os valores de x para os quais:

$$\begin{vmatrix} 5x-1 & 8x+4 \\ x-2 & -2x-1 \end{vmatrix} = 0 \Rightarrow (5x-1)(-2x-1) - (x-2)(8x+4) = 0 \Rightarrow$$

$$-10x^2 - 5x + 2x + 1 - 8x^2 - 4x + 16x + 8 = 0 \Rightarrow -18x^2 + 9x + 9 = 0 \Rightarrow$$

$$2x^2 - x - 1 = 0$$

$$r_{1,2} = \frac{+1 \pm \sqrt{1^2 - 4 \times 2 \times (-1)}}{2 \times 2} = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} \begin{cases} r_1 = 1 \\ r_2 = -\frac{1}{2} \end{cases}$$

Solução: $x = 1$ ou $x = -\frac{1}{2}$

3. Resolver o sistema, utilizando matrizes:

$$\times \frac{1}{4} \begin{bmatrix} 4 & 2 & -3 & | & -1 \\ 0 & -3 & 2 & | & 0 \\ 2 & 4 & -2 & | & 4 \end{bmatrix} \Rightarrow \begin{matrix} \times (-2) \\ + \\ \end{matrix} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{3}{4} & | & -\frac{1}{4} \\ 0 & -3 & 2 & | & 0 \\ 2 & 4 & -2 & | & 4 \end{bmatrix} \Rightarrow$$

$$\times \left(-\frac{1}{3}\right) \begin{bmatrix} 1 & \frac{1}{2} & -\frac{3}{4} & | & -\frac{1}{4} \\ 0 & -3 & 2 & | & 0 \\ 0 & 3 & -\frac{1}{2} & | & \frac{9}{2} \end{bmatrix} \Rightarrow \begin{matrix} + \\ \times \left(-\frac{1}{2}\right) \\ \times (-3) \\ + \\ \end{matrix} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{3}{4} & | & -\frac{1}{4} \\ 0 & 1 & -\frac{2}{3} & | & 0 \\ 0 & 3 & -\frac{1}{2} & | & \frac{9}{2} \end{bmatrix} \Rightarrow$$

$$\times \frac{2}{3} \begin{bmatrix} 1 & 0 & -\frac{5}{12} & | & -\frac{1}{4} \\ 0 & 1 & -\frac{2}{3} & | & 0 \\ 0 & 0 & \frac{3}{2} & | & \frac{9}{2} \end{bmatrix} \Rightarrow \begin{matrix} + \\ + \\ \times \frac{2}{3} \\ \times \frac{5}{12} \\ \end{matrix} \begin{bmatrix} 1 & 0 & -\frac{5}{12} & | & -\frac{1}{4} \\ 0 & 1 & -\frac{2}{3} & | & 0 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

Solução: $x=1$; $y=2$; $z=3$

$$= \frac{(x+1) \times (2(x^2 + 2x \cdot \Delta x + \Delta x^2) - 3) - (x + \Delta x + 1) \times (2x^2 - 3)}{(x+1) \times (x + \Delta x + 1)} =$$

$$= \frac{2x^3 + 4x^2 \Delta x + 2x \Delta x^2 - 3x + 2x^2 + 4x \Delta x + 2\Delta x^2 - 3 - 2x^3 + 3x - 2x^2 \Delta x + 3\Delta x - 2x^2 + 3}{(x+1) \times (x + \Delta x + 1)}$$

$$= \frac{2x^2 \Delta x + 2x \Delta x^2 + 4x \Delta x + 2\Delta x^2 + 3\Delta x}{(x+1) \times (x + \Delta x + 1)} = \frac{\Delta x \times (2x^2 + 2x \Delta x + 4x + 2\Delta x + 3)}{(x+1) \times (x + \Delta x + 1)}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{2x^2 + 2x \Delta x + 4x + 2\Delta x + 3}{(x+1) \times (x + \Delta x + 1)}$$

$$\frac{dy}{dx} = \frac{2x^2 + 4x + 3}{(x+1)^2}$$